**Example. 2.** Calculate the resistance of a circular block of material of specific resistance $10^{-6}$ ohm-m, radius of 1 cm and 10 cm in length.

**Solution.**
$$R = \rho \frac{l}{A}$$
$$= 10^{-6} \times \frac{10 \times 10^{-2}\,\text{m}}{\pi(10^{-2})^2} = \frac{10^{-3}}{\pi}\ \text{ohms}$$

**Example. 3.** A metallic wire is stretched to double its length. Find the ratio of the final and initial resistance of the wire.

**Solution.** Suppose the initial length of wire is $l'$ and its, area is $A$. On stretching its new length is $l'$ and area is $A'$.

Remember the volume of wire does not change. So,
$$Al = A'l'$$
given
$$l' = 2l$$
$\therefore$
$$A' = \frac{Al}{2l} = \frac{A}{2}$$
If the specific resistance of wire material is ρ, then the resistance of wire before stretching is
$$R = \rho \frac{l}{A}$$
And the resistance of wire after stretching is
$$R' = \rho \frac{l'}{A}$$
or
$$R' = \rho \frac{2l}{A/2}$$
Simplifying
$$R' = 4\frac{\rho l}{A} = 4R$$
or
$$\frac{R'}{R} = 4$$
$\therefore$
$$R' : R = 4 : 1$$

## 3.6 COMBINATION OF RESISTANCE

We can combine the resistance generally by two ways in any electrical circuit.

**(1) Series Combination :** In this connection resistances are joined by end to end. Second end of first resistance is joined to the first end of second resistance, etc. In this connection current flowing in all the resistances remains same but the potential difference between their ends may vary.

Suppose there are $R_1$, $R_2$ and $R_3$ resistances in series and $V_1$, $V_2$ and $V_3$ are corresponding voltages across these resistances. Say $I$ current is flowing in all three resistances.

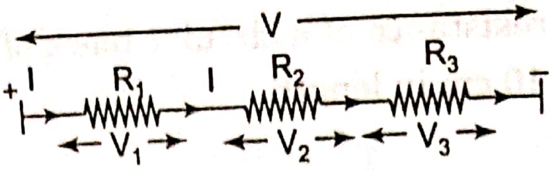


Fig. 3.4

Now according to Fig. 3.4, if $V$ is total voltage across the whole circuit then

$$V = V_1 + V_2 + V_3 \qquad ...(3.3)$$

We know by Ohm's law that

$$V_1 = IR_1,\ V_2 = IR_2,\quad V_3 = IR_3$$

and $$V = IR, \text{ say}$$

where $R$ is the net resistance of the circuit.

Then from Eq. (3.3)

$$IR = IR_1 + IR_2 + IR_3$$

or $$\boxed{R = R_1 + R_2 + R_3} \qquad ...(3.4)$$

Thus in series combination the equivalent resistance is sum of all the resistances joined in series.

**(2) Parallel Combination :** In parallel combination first end of all the resistances (To be joined) are connected to one point and second end of all the resistances are connected to the another point in the circuit (Fig. 3.5).

In this connection potential difference between ends of all resistances remain same but current varies

Here, $$I = I_1 + I_2 + I_3 \qquad ...(3.5)$$

Using Ohm's law ($V = IR$) in Eq. (3.5)

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \qquad ...(3.6)$$

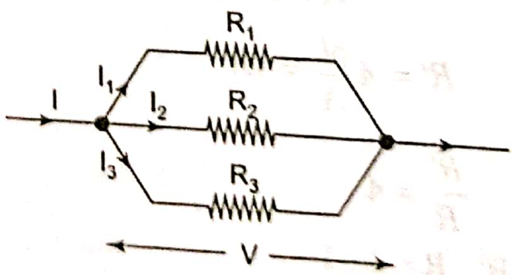


Fig. 3.5

Where $R$ is the net resistance of the combination.

Solving Eq. (3.6)

$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \qquad ...(3.7)$$

Thus, the reciprocal of the net resistance of the resistances connected in parallel is equal to the sum of the reciprocal of all resistances.

In this case the net resistance is smaller than the smallest resistance among the connected resistance.

## 3.7 KIRCHHOFF'S LAW

It is used to determine the distribution of current among the different parts of the circuit. The law state that

(i) The algebraic sum of currents meeting at any junction in the circuit is zero.

$$\sum i = 0$$

For example (Fig. 3.8) there are $i_1, i_2, i_3, i_4$ and $i_5$ currents respectively, in the five conductors. For conventionally the incoming currents $i_1, i_3$ towards junction is taken positive and currents going away, *i.e.*, $i_2, i_4, i_5$ are taken negative. So

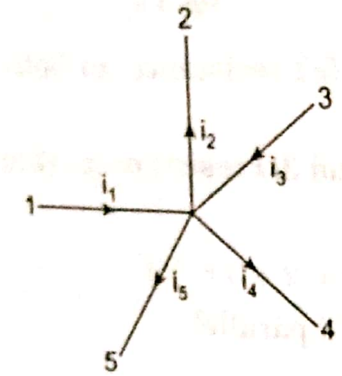


Fig. 3.8

$$i_1 - i_2 + i_3 - i_4 - i_5 = 0$$

or

$$i_1 + i_3 = i_2 + i_4 + i_5$$

The first law also expresses the conservation of charge.

(ii) Algebraic sum of all potential differences along a closed loop in any circuit is zero.

In any closed loop of a circuit the algebraic sum of the products of current and resistances in each part of the loop is equal to the algebraic sum of all the emf 's in that loop.

$$\sum iR = \sum E$$

It is the law of conservation of energy.

**Note:** When you are moving in the direction of current,† the direction of current will be taken positive and *emf* is taken positive when traverse from negative to positive pole of battery.

Now look this Fig. 3.8, let's move in Anti-clockwise direction.

In Fig. 3.9 $i_1$ is taken positive while $i_2$ is taken negative for loop 1.

The equation becomes

$$i_1R_1 - i_2R_2 = E_1 - E_2 \qquad \text{...(3.8)}$$

$$(\because V_1 - V_2 = E_1 - E_2)$$

*For loop 2*

$$i_2R_2 + (i_1 + i_2)R_3 = i_2E_2 \qquad \text{...(3.9)}$$

$$[\because i_3 = i_1 + i_2;\ 1^{st}\ \text{law}]$$

† *It is your choice to choose the direction either clockwise or anticlockwise but remember it must be in the same directional for all the meshes of a circuit.*

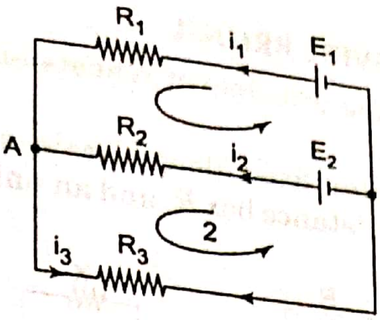


Fig. 3.9

## 3.8 WHEATSTONE BRIDGE

It is used to find out the unknown resistance of a conductor.

In this arrangement four resistances are so connected as to form a parallelogram. In Fig. 3.10, *ABCD* is a parallelogram having resistance *P*, *Q*, *R* and unknown *S*. *BD* is connected with a galvanometer *G* and *AC* is connected by a cell *E*.

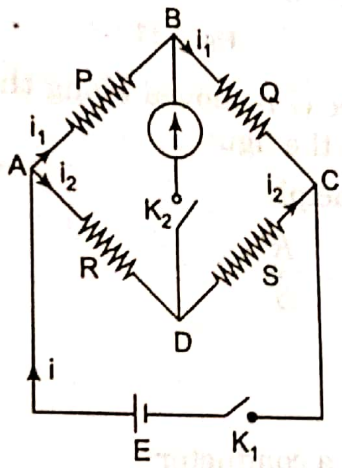


Fig. 3.10

*P*, *Q*, *R* and *S* are so adjusted that on passing $K_2$ after key $K_1$ there is no deflection in the G. This means that there is no current in the diagonal *BD*. This type of situation in which there is no current flow through galvanometer, the bridge is said to be balanced.

Applying Kirchhoff 's [*Krc*] II[nd] law for closed loop *ABDA*

$$i_1P - i_2R = 0 \quad ...(3.10)$$

Similarly for *BCDB*

$$i_1Q - i_2S = 0 \quad ...(3.11)$$

Solving Eq. 3.10 and 3.11

$$\frac{i_1P}{i_1Q} = \frac{i_2R}{i_2S}$$

So,

$$\boxed{\frac{P}{Q} = \frac{R}{S}}$$

This is the expression to find out the unknown resistance by the Wheatstone bridge.

The point is to be noted that if all the four resistances are of the *same order* then the ... will be maximum.

## 3.9 METER BRIDGE OR SLIDE WIRE BRIDGE

Meter bridge is a device based on the principle of Wheatstone bridge for the determination of the resistance of a conductor.

$AC$ is a 1 metre wire of constantan fixed along a scale on wooden box. $AC$ is joined by a $L$ shaped two thick copper strips. A resistance box $R$, and an unknown resistance wire (conductor) is fixed like the Fig. 3.11.

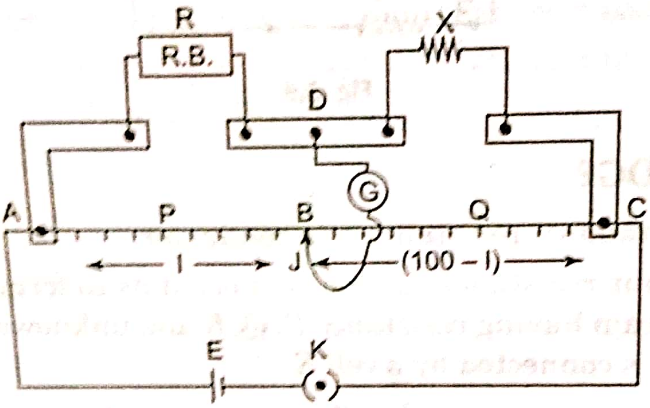


Fig. 3.11

Jockey $J$ through a galvanometre $G$ is moved along the wire. $B$ is the null point†. All the other arrangements are according to the figure.

Applying Wheatstone bridge principle

$$\frac{P}{Q} = \frac{R}{S}$$

If length of $AB$ is $l$ cm

Then length of $BC = (100 - l)$ cm

We know, resistance ∝ length for a conductor.

So,
$$\boxed{\frac{l}{100 - l} = \frac{R}{S}}$$

$$S = (100 - l)\frac{R}{l}$$

also. ...useful for complicated circuits

**Example. 6** Find out the current in both the meshes (closed loop) of the following circuit.

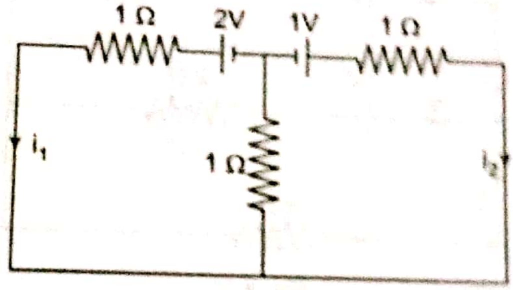


Fig. 3.13

**Solution.** Applying Krc II[nd] law in both loops

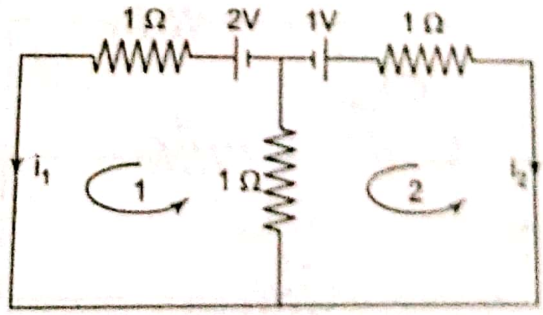


Fig. 3.13 a

$$i_1 \times 1 + (i_1 + i_2) \times 1 = 2 \qquad \text{...(3.12)}$$

$$i_2 \times 1 + (i_1 + i_2) \times 1 = 1 \qquad \text{...(3.13)}$$

Solving these two equations

$$i_1 = 1 \text{ amp}, \quad i_2 = 0 \text{ amp}$$

Now replace a battery having emf more than 2V. Say 3V and find the value and direction of $i_2$. Discuss the output with your classmates.

## 3.11 ELECTRIC POWER

When an electric current flows through a conductor, electrical energy is used up and we say that current is doing work. We know that rate of doing work is called power. So electrical power is the electrical work done per unit time. It is defined as the rate at which electrical energy is dissipated into other forms of energy is called electric power $P$.

$$\therefore \quad P = \frac{\text{Work done energy dissipated}}{\text{time}}$$

$$\therefore \quad P = \frac{W}{t} \qquad \text{...(3.22)}$$

The unit of power is Joule/sec or watt (W).

$$1 \text{ watt} = 1 \text{ Joule/sec.}$$

We know that work done in taking a charge of $q$ under the p.d. $V$ is given by

$$W = Vq$$

And also

$$q = it$$

where, $i$ in the current in the wire and $t$ is the time.

or from Eq. 3.22

$$P = V\,i \text{ watt} \qquad \text{...(3.23)}$$

Since, $V = i\,R$, $R$ is the resistance offered by the conductor in current flow then

$$P = i^2R \qquad \text{...(3.24)}$$

Now combining the both power equations.

$$P = V\,i = i^2R$$

$$= \frac{V^2}{R} \qquad \text{...(3.25)}$$

Watt is a small unit so some bigger units are used. In domestic or commercial purposes kilowatt unit is used

$$1 \text{ kilowatt} = 1000 \text{ watts}$$

In mechanics, the unit of power is often written in horse—power

$$1 \text{ horse power (H.P.)} = 746 \text{ watt}$$

Actually, when work is done, an equal amount of energy is consumed. So we can say that electrical power is the rate at which electrical energy is consumed. When an electrical appliance, consumes electrical energy at the rate of 1 Joule per second, its power is said to be 1 watt.

## 3.12 ELECTRIC ENERGY

Electric energy is the total work done by an electric current in a given time. Actually it is equal to the total energy consumed in an electric circuit in a given time.

SI unit of electrical energy is joule. The commercial unit of electric energy is kilowatt-hour (kW-h) or Board of Trade Unit (B.T.U.) One kilowatt-hour is the amount of electrical energy consumed when an electrical appliance having a power rating of 1 kilowatt is used for 1 hour.

$$1 \text{ kW-h} = 1 \text{ kilowatt} \times \text{hour}$$

$$= 1000 \text{ watt} \times 60 \times 60 \text{ sec}$$

$$= 3.6 \times 10^6 \text{ watt-sec}$$

$$\therefore \quad 1 \text{ kW-h} = 3.6 \times 10^6 \text{ W-s} = 3.6 \times 10^6 \text{ J}$$